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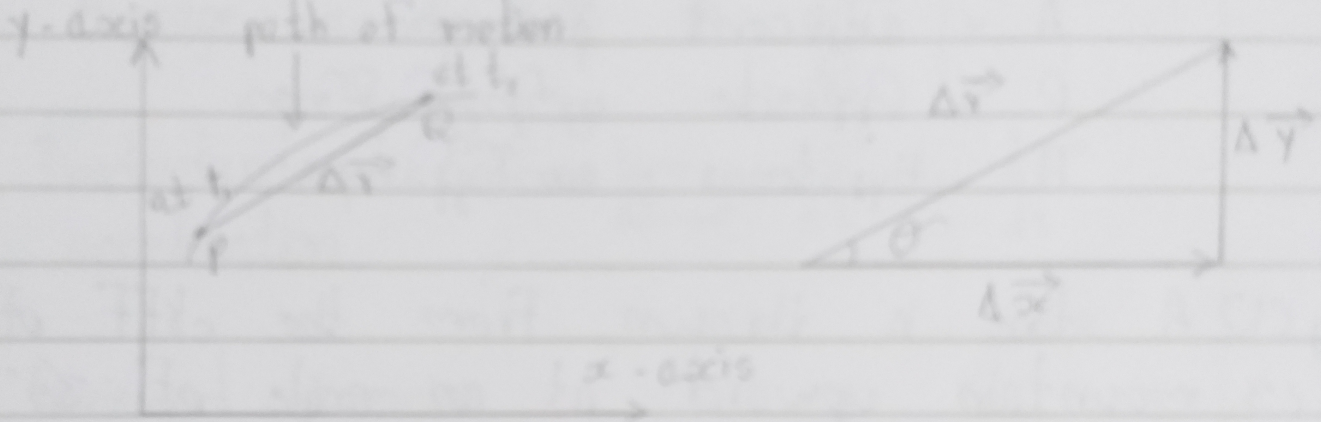
Chapter (1)
Motion in a plane

1.1 Two-Dimensional motion



Motion of an object in two dimensions

$$\Delta\vec{r} = \Delta\vec{x} + \Delta\vec{y}$$

$$\Delta r^2 = \Delta x^2 + \Delta y^2$$

$$\boxed{\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}}$$ (The magnitude of $\Delta\vec{r}$)

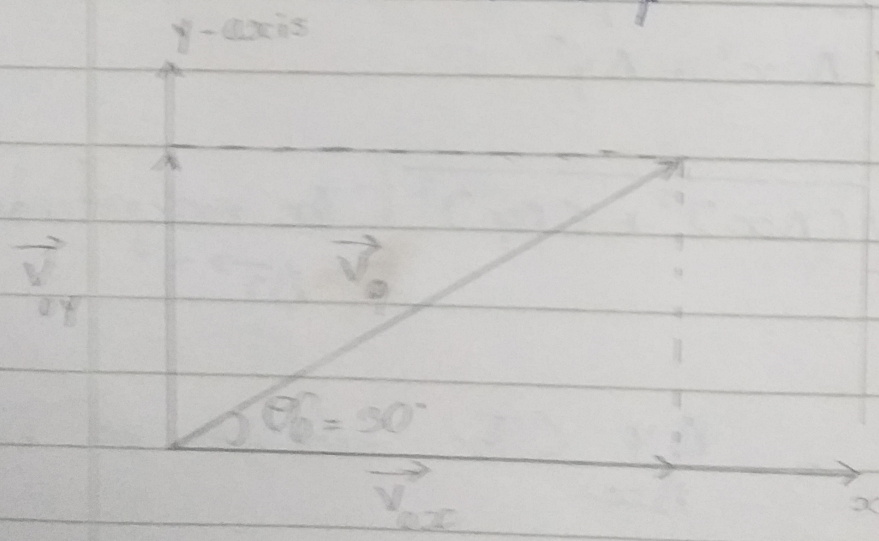
$$\tan\theta = \frac{\Delta y}{\Delta x}$$ (The direction of $\Delta\vec{r}$)

$$\boxed{\theta = \tan^{-1}\frac{\Delta y}{\Delta x}}$$

sin	cos	tan	cot	sec	cosec
O	A	O	A	H	H
H	H	A	O	A	O

A = adjacent နီကပ်ဘုရား
O = Opposite မျက်နှာချင်းဆိုင်ဘုရား
H = Hypotenus ထောင့်မှန်ခံဘုရား

Eg (1) 7.6.23 A stone is thrown from the cliff of a mountain upward at an angle of 30° to the horizontal with an initial speed of 20 ms^{-1}. Calculate the x and y components of its initial velocity



horizontal components of initial velocity

$V_{ox} = V_0 \cos\theta_0 = 20 \cos$

$= 20 \times 0.866 = 17.32$

Vertical components of initial velocity

$\cancel{V_{oy} = V_0 \sin\theta_0}$

$\frac{V_{oy}}{V_0} = \sin\theta$

$V_{oy} = V_0 \sin\theta$

$V_{oy} = 20 \times 0.5 = 10 ms^{-}$

Average velocity $= \vec{V} = \frac{\Delta \vec{r}}{\Delta t}$

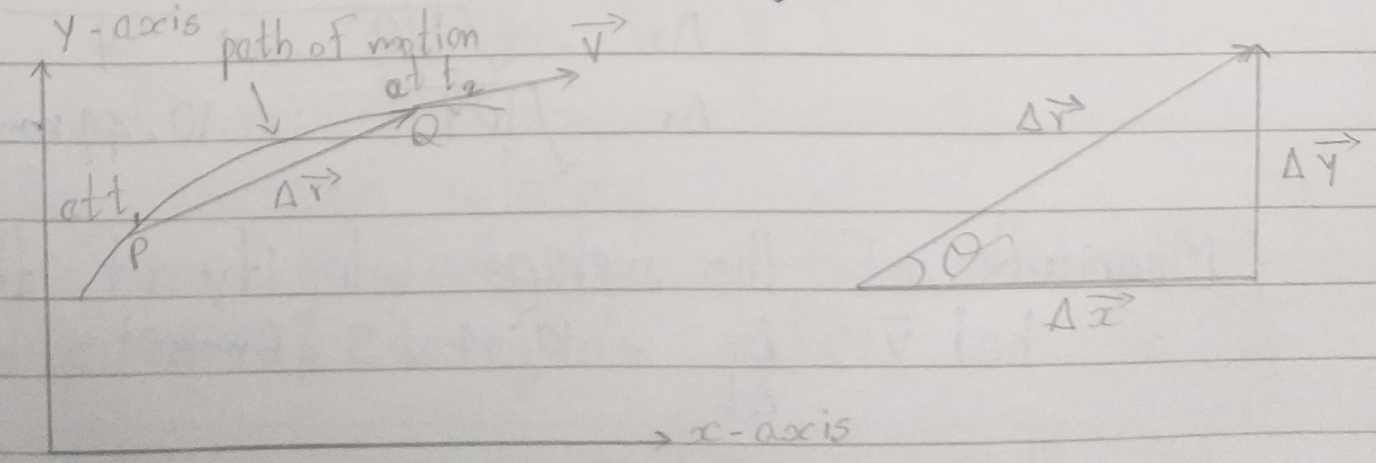
Instantaneous velocity $= \vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

Average acceleration $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

Instantaneous acceleration $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

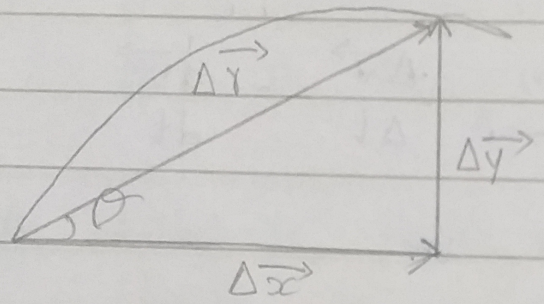
Reviewed Exercise

Draw the diagram to show the direction of velocit of an object which is moving along a curve path and then draw the vector diagram to show the velocity components in x and y directions at a starting point P.



Dream it. Do it.

Eg(1.2) A soccer ball is kicked at an angle with the ground. The ball traverse the horizontal distance of 10m and the vertical distance of 3m in 3s. Find the displacement and the average velocity of the ball in 3s.



the horizontal distance $\Delta x = 10m$

the vertical distance $\Delta y = 3m$, $\Delta t = 3s$

the magnitude of displacement $= \Delta r$

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\Delta r = \sqrt{(10)^2 + (3)^2}$$

$$\Delta r = \sqrt{100 + 9}$$

$$\Delta r = \sqrt{109} = 10.44m$$

Magnitude of the average velocity of the bal $\bar{v} = \frac{\Delta r}{\Delta t} = \frac{10.44}{3} = 3.48 ms^{-1}$

$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{3}{10} = 0.3$$

$\tan \theta = 0.3000$

$\tan \theta = \tan 16^\circ 42'$

$\theta = 16^\circ 42'$

$\theta = 16.7^\circ$

Defs:

1. Displacement

Displacement is the distance travelled along a particular direction.

2. The average speed

The average speed is the ratio of total distance to time taken.

$$V_{av} = \frac{s}{t} \text{ (or) } \bar{v} = \frac{s}{t}$$

3. The average velocity

The average velocity is the ratio of total displacement to time taken.

$$\vec{V}_{av} = \frac{\vec{s}}{t} \text{ (or) } \vec{\bar{v}} = \frac{\vec{s}}{t}$$

4. The average acceleration
The average acceleration is the ratio of the change in velocity to the time taken.

$$\vec{a}_{av} = \frac{\vec{v} - \vec{v}_0}{t}$$

5. The instantaneous speed
The instantaneous speed is defined as the time rate of change of distance.

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

6. The instantaneous velocity
The instantaneous velocity is defined as the time rate of change of displacement.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$$

7 The instantaneous acceleration

The instantaneous acceleration is defined as the time rate of change of velocity.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

8 Two-dimensional motion

The object may move in both the x and y direction simultaneously. The motion of the object is said to be a two-dimensional motion.

2 Projectile Motion

Projectile

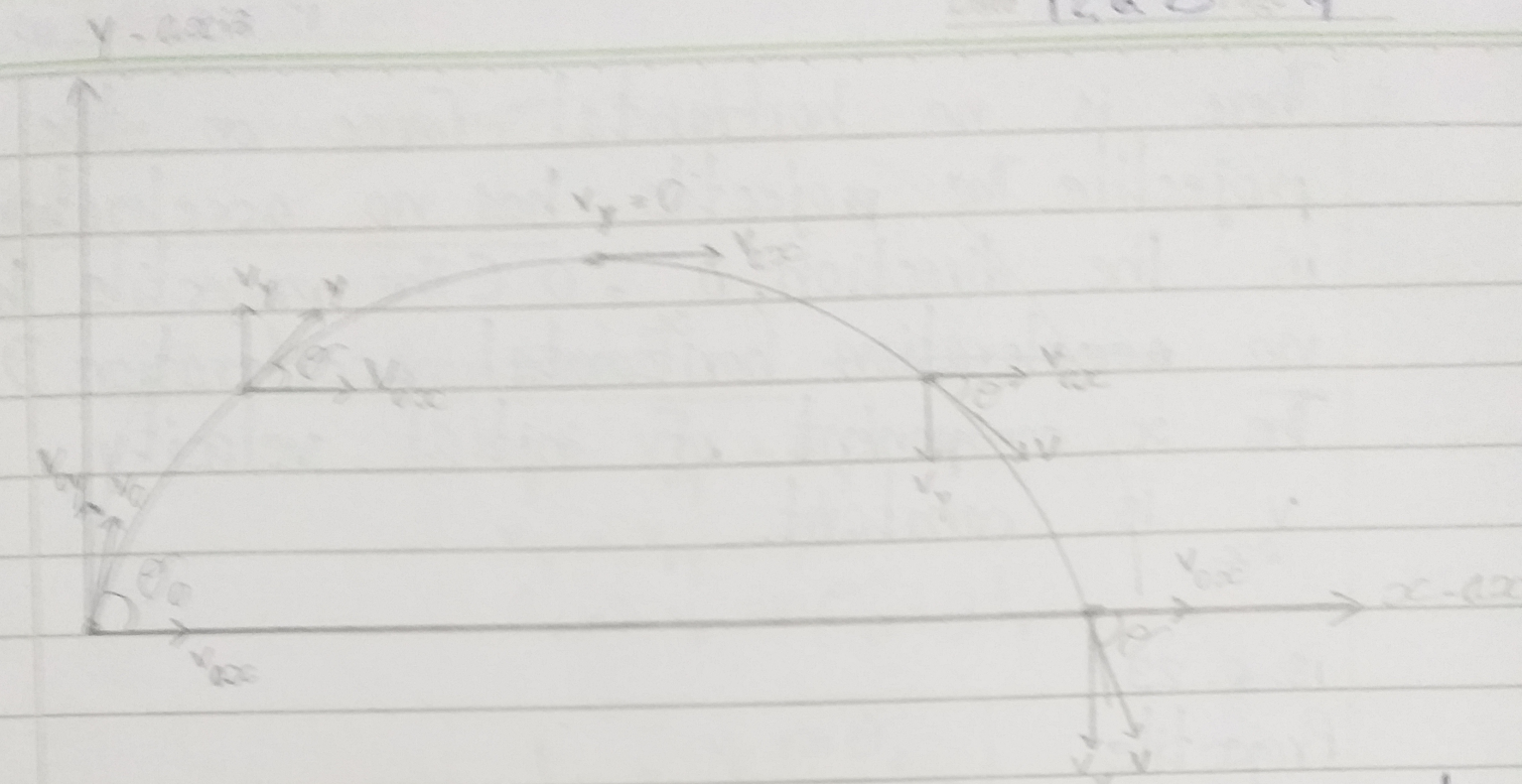
A projectile is any object thrown into space upon which the only acting force is the gravity.

Trajectory

The path followed by a projectile is known as a trajectory.

Projectile Motion

The motion of an object moving in both horizontal x direction and vertical y direction simultaneously is called the projectile motion (Examples for the projectile motion are water fountain, the motion of cannon ball, the motion of the Football ~~as shown~~



1. The initial velocity of projectile is v_0. It makes an angle with the horizontal.
2. The projectile with have two motions, the horizontal motion (x direction) and the vertical motion (y direction)
3. The horizontal component of initial velocity

$$v_{0x} = v_0 \cos\theta_0$$

The vertical component of initial velocity

$$v_{0y} = v_0 \sin\theta_0$$

4. Only the gravitational force acts on the projectile, The direction of this force is downward. Therefore, the projectile has an acceleration along the y-axis, a_y. This acceleration is due to gravity, g. ($a_y = -g$).

5. There is no horizontal force on the projectile. The projectile has no acceleration in the direction, $a_x = 0$. (The projectile has no ~~acceleration~~ horizontal acceleration.)
The x component of initial velocity v_{0x} is constant.

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Projectile
The magnitude of the velocity,

$$V = \sqrt{v_x^2 + v_y^2}$$

The direction of the velocity,

$$\tan\theta = \frac{v_y}{v_x}$$

Vertical component,

$$v_y = 0$$

The time to reach highest point,

$$t_1$$

Dream it. Do it.

The maximum height,
H

From horizontal,

$$v_y = v_{oy} + a_y t$$
$$v_y = v_{oy} - gt$$
$$0 = v_0 \sin\theta_0 - gt_1$$

$$\boxed{t_1 = \frac{v_0 \sin\theta_0}{g}}$$

The time of flight . T

The time of the flight is twice the time to reach the maximum height

$T = 2t_1$

$$T = \frac{2 V_0 \sin\theta_0}{g}$$

$$H = \frac{(V_0 \sin\theta_0)^2}{2g}$$

	Linear motion	x-direction	y-direction
1.	$v = v_0 + at$	$v_x = v_{0x} + a_x t$	$v_y = v_{0y} + a_y t$
2.	$v^2 = v_0^2 + 2as$	$v_x^2 = v_{0x}^2 + 2a_x x$	$v_y^2 = v_{0y}^2 + 2a_y y$
3.	$s = v_0 t + \frac{1}{2}at^2$	$x = v_{0x} t + \frac{1}{2}a_x t^2$	$y = v_{0y} t + \frac{1}{2}a_y t^2$

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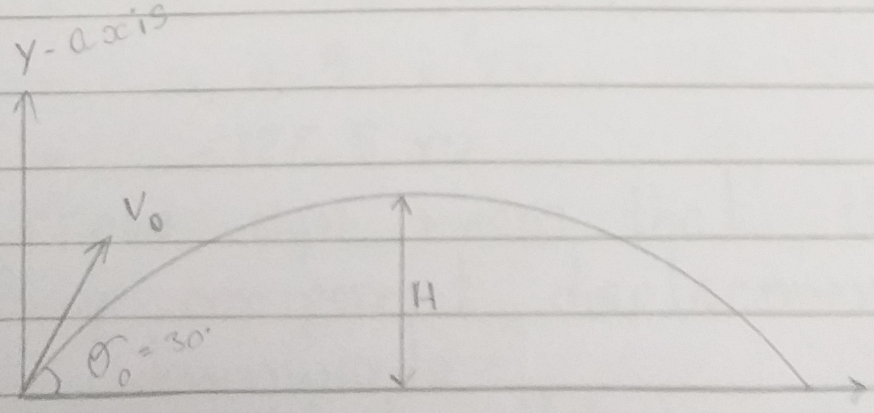
Eg. 1.3 An object is projected upward with a 30° launch angle an an initial speed of 40 ms^{-1}. How long will it take for the object to reach the top of its trajectory? Find the maximum height of its trajectory.



$V_0 = 40\ ms^{-1}$

$\theta_0 = 30°$

$t_1 = ?, H = ?$

$a_y = -g, g = 9.8\ ms^{-2}$

$V_{0y} = V_0 \sin\theta_0$

$= 40 \times 0.5$

$= 20$

$t_1 = \frac{V_0 \sin\theta_0}{g} = \frac{20}{9.8} = 2.04\ s$

$H = \frac{(V_0 \sin\theta_0)^2}{2g} = \frac{(20)^2}{2 \times 9.8} = \frac{400}{19.6} = 20.4\ m$

Dream it. Do it.

Eg 1.4 A bomb is dropped from an airplane moving horizontally with its speed of $50\ ms^{-1}$. If the bomb will reach the ground in 5 s, Find the ~~alt~~ altitude of the plane. The air ~~resistone~~ resistance is negligible.

$v_{0x} = 50\ ms^{-1}$

$v_{0y} = 0$

$t = 5\ s$

$a_y = -g$

$g = -9.8\ ms^{-2}$

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$= 0 \times 5 + \frac{1}{2}(-g) \times 5^2$$

$$= 0 + \frac{1}{2}(9.8) \times 25$$

$$= 4.9 \times 25$$

$$= -122.5\ m$$

Minus sign means that the direction of y component displacement is downwards.

The ~~altitude~~ altitude of the plane is 1225 m.

Dream it. Do it.

Eg 1.5 A stone is thrown with a speed $20\ ms^{-1}$ and at an angle 30° above the horizontal. Find
(i) horizontal range
(ii) maximum height reached
(iii) the time of flight of the stone.

(ii) Maximum height $H = ?$

$g = 9.8\ ms^{-2}$, $V_0 = 20\ ms^{-1}$, $\theta_0 = 30°$

$$H = \frac{(V_0 \sin\theta_0)^2}{2g}$$

$$H = \frac{(20 \times 0.5)^2}{2 \times 9.8}$$

$$H = \frac{400 \times 0.25}{19.6}$$

$$H = \frac{100}{19.6} = 5.10\ m$$

Dream it. Do it.

(iii) the time of Flight of the stone T = ?

$g = 9.8\ ms^{-2}$, $V_0 = 20\ ms^{-1}$, $\theta_0 = 30^\circ$

$$T = \frac{2 \times V_0 \sin\theta_0}{g}$$

$$T = \frac{2 \times (20 \times 0.5)}{9.8}$$

$$T = \frac{2 \times 10}{9.8} = \frac{20}{9.8}$$

$$= 2.04\ s$$

(i) horizontal range $R = ?$

$g = 9.8\ ms^{-2}$, $V_0 = 20\ ms^{-1}$, $\theta_0 = 30^\circ$

$$R = \frac{V_0^2 \sin 2\theta_0}{g}$$

$$R = \frac{(20)^2 \sin 60^\circ}{9.8} = \frac{400 \times 0.8660}{9.8}$$

$$R = \frac{346.4}{9.8} = \frac{3464}{98}$$

$$= 35.35\ m$$

Dream it. Do it.

Ex 4. A footballer kicked a ball with an initial velocity of $20 ms^{-1}$ at an angle of 60°.

(i) How long is the ball in the air?

(ii) What are the range and maximum height of the ball?

$V_0 = 20 ms^{-1}$, $\theta = 60^\circ$, $g = 9.8 ms^{-2}$

(i) $T = ?$

$$T = \frac{2 V_0 \sin \theta_0}{g}$$

$$= \frac{2(20) \sin 60^\circ}{9.8} = \frac{40 \times 0.8660}{9.8}$$

$$= 3.53 \text{ s}$$

(ii) $H = ?$

$$H = \frac{(V_0 \sin \theta_0)^2}{2g}$$

$$= \frac{(20 \times \sin 60^\circ)^2}{2 \times 9.8}$$

$$= \frac{(20)^2 (0.8660)^2}{19.6} = \frac{400 \times 0.9050}{19.6}$$

$$= 15.3 \text{ m}$$

Dream it. Do it.

R = ?

$R = V_{0x} T$

$= V_0 \cos\theta^\circ \, T$

$= 20 \cos 60^\circ \times 3.53$

$= 20 \times 0.5 \times 3.53$

$= 10 \times 3.53$

$= 35.3 \text{ m}$

Circular Motion

Circular motion is a movement of an object along the circumference of a circle (or) rotation along a circular path.
Circular motion is a specific type of a two-dimensional motion.

The angular displacement

The angular displacement is ~~the~~ the ratio of the arc length to radius.

1 rad

The magnitude of angular displacement θ, measured in radian (rad), is given by

$$\theta = \frac{s}{r}$$

where,
θ = angular displacement
s = arc length
r = radius of circle

Draw it. Do it.

One radian is defined as the angle subtended at the centre of a circle by an arc whose length is equal to the radius of a circle.

If $s = r$, $\theta = 1$ rad

1 rev = 360° = 2π rad

For example, $60° = \frac{\pi}{180°} \times 60° = \frac{\pi}{3}$ rad

$$30° = \frac{\pi}{180°} \times 30° = \frac{\pi}{6} \text{ rad}$$

Average Angular Velocity

Average Angular velocity is the ratio of the change in angular displacement to time taken.

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$ where,

$\bar{\omega}$ = average angular velocity,

$\Delta\theta$ = angular displacement

Δt = time taken (time interval)

Dream it. Do it.

Instantaneous Angular velocity

Instantaneous angular velocity is defined as the time rate of change of angular displacement.

At constant angular velocity,

$$\bar{\omega} = \omega, \quad \omega = \frac{\theta}{t}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

The Average Angular acceleration

The average angular acceleration is the ratio of change of angular velocity to time taken.

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

where,

$\bar{\alpha}$ = average angular acceleration

ω_1 = angular velocity at time instant t_1

ω_2 = angular velocity at time instant t_2

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x3. A ball is thrown horizontally with an initial speed of $10 ms^{-1}$ from an 80 m cliff. How long does it take to reach the ground?

$v_{ox} = 10 ms^{-1}$

$y = -80 m$

$-g = -9.8 ms^{-2}$, $v_{oy} = 0$

$t = ?$

$y = v_{oy} t + \frac{1}{2} a_y t^2$

$-80 = 0 \times t + \frac{1}{2}(-g) t^2$

$= 0 + \frac{1}{2} \times (-9.8) t^2$

$-80 = 0 - 4.9 t^2$

$\frac{80}{4.9} = t^2$

$16.33 = t^2$

$t = \sqrt{16.33} = 4.04 s$

time to reach the ground = 4.04 s

Ex 2 A stone is thrown horizontally from a cliff 100 Ft high. The initial velocity is 20 fts^{-1}. How far from the base?

$v_0 = 20 Ft^{-1}$

$y = -100$ Ft (∵ downwards)

$x = ?$

$v_{oy} = 0$

$a_y = -g$

$g = 32 Fts^{-2}$

$$y = v_{oy}t + \frac{1}{2}a_y t^2$$

$$-100 = 0 \times t + \frac{1}{2}(-32)\, t^2$$

$$-100 = -16\, t^2$$

$$\frac{-100}{16} = t^2$$

$$\frac{10^2}{4^2} = t^2$$

$$t = 2.5 \text{ s}$$

$$x = v_{0x} = 20\ Fts^{-1}$$

$$x = v_{0x} t + \frac{1}{2} a_x t^2$$
$$x = v_{0x} t + 0$$
$$x = 20 \times 2.5\ ft$$
$$= 50\ ft$$

EX 1. A cannon ball is shot with initial velocity $141.4\ ms^{-1}$ with 45° angle of ~~incln~~ inclination Find the position and velocity of the cannon ball at 5s.

$v_0 = 141.4\ ms^{-1}$

$\theta_0 = 45^\circ$

$t = 5\ s$

~~a~~ $a_y = -g = -9.8\ ms^{-2}$

~~$v_{0y} = 0$~~

$x = ?\ ,\ y = ?$

$v = ?\ ,\ \theta = ?$

$v_x = v_{ox} + a_x t$

$v_x = v_{ox}$

$= v_0 \cos\theta_0$

$= 141.4 \times \cos 45°$

$= 141.4 \times 0.7071$

$= 99.97$ m

$v_y = v_{oy} + a_y t$

$= v_0 \sin\theta_0 + (-g)t$

$= 141.4 \times \sin 45° + (-9.8 \times 5)$

$= 99.97 - 49$

$= 50.97$ m

$x = v_{ox} t$

$= 99.97 \times 5$

$= 499.85$ m

~~$y = v_{oy} t + \frac{1}{2} a_y t^2$~~

~~$= 0 \times 5 + \frac{1}{2}(-9.8) 5^2$~~

~~$= -4.9 \times 25$~~

$$y = v_{oy} t + \frac{1}{2} a t^2$$

$$= 99.97 \times 5 + \frac{1}{2}(-9.8)5^2$$

$$= 499.85 + (-4.9) \times 25$$

$$= 499.85 - 122.5$$

$$= 377.35 \text{ m}$$

The magnitude of velocity,

~~$v = v_0 + at$~~

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{(99.97)^2 + (50.97)^2}$$

$$= \sqrt{(9994) + (2598)}$$

$$= \sqrt{12592} = 112.2 \text{ ms}^{-1}$$

The direction of velocity,

$$\tan\theta = \frac{v_y}{v_x}$$

$$= \frac{50.97}{99.97} \quad \text{~~2598~~}$$

$$\tan\theta = \tan 27^\circ 1'$$

$$\theta = 27^\circ 1'$$

When a body is moving with a constant angular acceleration $\bar{\alpha} = \alpha$.

can be written as

$$\bar{\alpha} = \alpha = \frac{\omega - \omega_0}{t}$$ where

α = angular acceleration,
ω = Final angular velocity,
ω_0 = initial angular velocity,
t = time taken,

Relation between Angular and Linear Quantities

Tangential speed

where r is constant

$$v = r\omega$$

Dream it. Do it.

Eg 1.6) Express angular velocity of a 45 rpm (revolutions per minute) record turntable in units of rad s^{-1}.

$$1 \text{ rev min}^{-1} = \frac{360^\circ}{60} = \frac{1 \text{ rev}}{1 \text{ min}}$$

$$= \frac{2\pi}{60} = \text{rads}^{-1}$$

$$45 \text{ rev min}^{-1} = \frac{45^{3} \times 2\pi}{60_{4}^{2}} = \frac{3\pi}{2} = 1.5\pi \text{ rads}^{-1}$$

Eg 1.7) The angular velocity of a ~~rod~~ rotating disc increases from 2 rad s^{-1} to 5 rads^{-1} in 10 s. $\bar{\alpha}$ = ?

$\omega_0 = 2 \text{ rad s}^{-1}$, $\omega = 5 \text{ rads}^{-1}$ = $t = 10$s

The average angular acceleration

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{5 - 2}{10} = \frac{3}{10}$$

$$= 0.3 \text{ rad s}^{-2}$$

Dream it. Do it.

Eg (1.8) A particle in disc rotating with a uniform angular speed of 2 rev s^{-1} is 0.2 m From the axis of rotation. What are (i) the tangential speed of the particle and (ii) the angle through which it rotates in 3s?

$\omega = 2 \text{ rev } s^{-1} (\text{rps}) = 2 \times 2\pi = 4\pi \text{ rad } s^{-1}$

$r = 0.2 \text{ m}$

(i) The tangential speed, $V = r\omega$

$= 0.2 \times 4\pi = 0.8\pi$

$= 0.8 \times 3.142 = 2.514 \text{ m s}^{-1}$

(ii) The angle of rotation in $t = 3s$,

$\theta = \omega t = 4\pi \times 3$

$= 12\pi \text{ rad} = \frac{12\pi}{2\pi} \text{ rev}$

$= 6 \text{ rev}$

Dream it. Do it.

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Exa. A disc rotating at angular speed of 10 rad s^{-1} is slowed down by a uniform angular acceleration to a speed of 4 $rads^{-1}$ in 3s. What is ~~average~~ angular a?

$\omega = 10 \text{ rad } s^{-1}$

$\omega^{o} = 4 \text{ rad } s^{-1}$

$t = 3s$

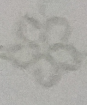
$\alpha = ?$

$$\alpha = \frac{\omega - \omega_0}{t}$$

The ~~average~~ angular acceleration

$$\alpha = \frac{4 - 10}{3} = \frac{-6}{3}$$

$$= -2 \text{ rad } s^{-2}$$



Dream it. Do it.

5. A circular disc rotates initially at rest experiences a uniform angular acceleration of 0.25 $rad\,s^{-2}$. What is angular speed after

$\omega_0 = 0\ rad\,s^{-1}$ ($\because$ rest)

$\alpha = 0.25\ rad\,s^{-2}$

$t = 10\ s$

$\omega = ?$

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\omega = \alpha t + \omega_0$$

$$= 0.25 \times 10 + 0$$

$$= 2.5\ rad\,s^{-1}$$

7. The tips of the blades in a food blender are moving with a constant speed of 21 ms^{-1} in a circle that has a radius of 0.053 m. How much time does take for the blades to make one revolution?

$v = 21\ ms^{-1}$

$r = 0.053\ m$

$\theta = 1\ rev$

$= 2\pi$

$t = ?$

$v = rw$

$w = \frac{v}{r}$

$w = \frac{21}{0.053} = 396.23$

$w = \frac{\theta}{t}$

$t = \frac{\theta}{w} = \frac{2\pi}{396.23} = \frac{2 \times 3.142}{396.23}$

$= 0.02\ s$

Dream it. Do it.

Chapter (2)
Rotational Dynamics

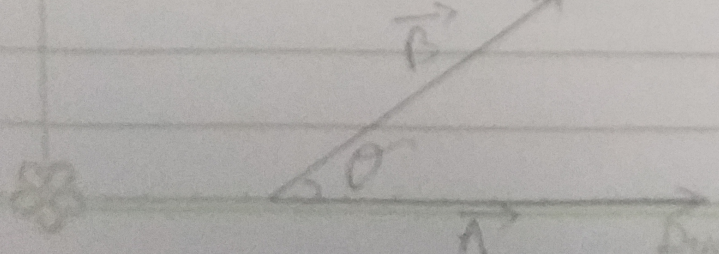
The dynamic of the rotational motion involves ~~deserb~~ describing the angular acceleration of an object in terms of its rotational inertia and the torques that act on it.

2.1 Scalar Product and Vector Product

- Vector can be multiplied in two different ways; the scalar product and the vector product.

Scalar Product (Dot Product)

* In the scalar product, a scalar can be formed by multiplying two vectors. The scalar product of vectors $\vec{A}$ and $\vec{B}$ is defined.



$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

where θ - the angle between $\vec{A}$ and $\vec{B}$.

A and B are - the magnitudes of $\vec{A}$ and $\vec{B}$.

* The result of scalar product (dot product) of two vectors is a scalar quantity. The scalar product is also called the dot product.

The scalar product has the commutative property.

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

Vector Product (Cross Product)

* The vector product of vectors $\vec{A}$ and $\vec{B}$ defined as

$$\vec{A} \times \vec{B} = \vec{C}$$

The result of vector product (cross product) of two vectors is a vector quantity.

* The magnitude of $\vec{C}$ is $C = AB \sin\theta$

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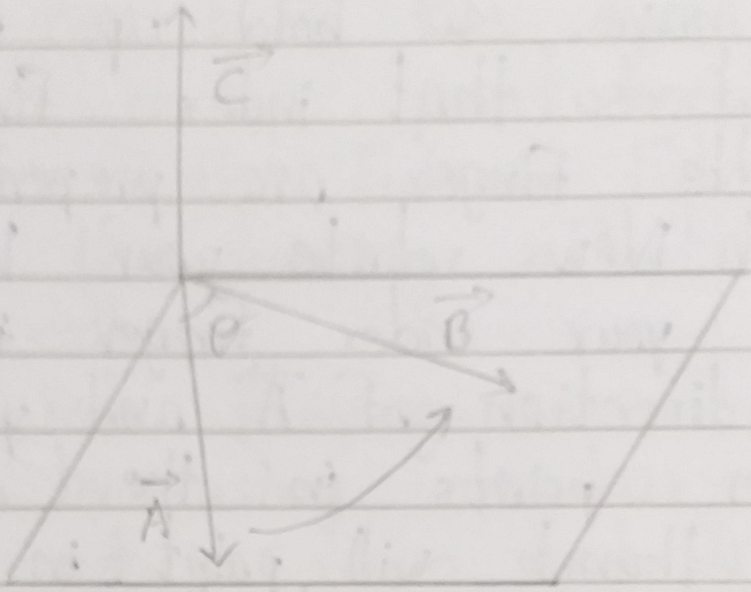
* The direction of $\vec{C}$ is perpendicular to the plane that contains $\vec{A}$ and $\vec{B}$



The direction of $\vec{C}$ is prependicular to both $\vec{A}$ and $\vec{B}$

The vector product does not obey the commutative property.

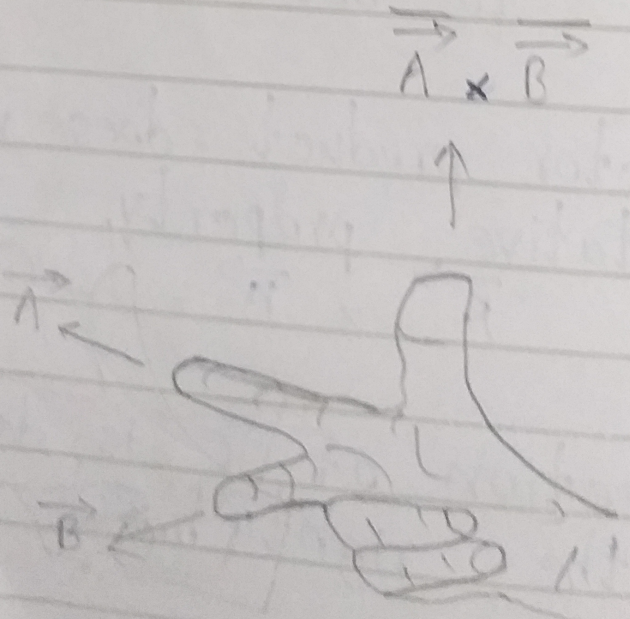
$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

* The direction of $\vec{C}$ is determined by using the right-hand rule.

Dream it. Do it.

Right - hand rule

To use the right - hand rule, first you have to hold up your right hand so that index finger and middle finger are prependicular to thumb. Now rotate your hand such that your index finger ~~in~~ t points in the direction of $\vec{A}$ and your middle finger points in the direction of $\vec{B}$. Your thumb will point in the direction of the cross product $\vec{A} \times \vec{B}$



Right - hand rule

2.2 Turning Effect of Force

When a force is applied to an object, it can turn the object ~~about~~ about a certain point known as the pivot (or) the ~~fulcrum~~ fulcrum.

This is ~~the~~ turning effect of force called

Examples: Kids playing seesaw, lifting a load by using crowbar, opening a door of a room and tightening (or) loosening a nut by turning a spanner.

Moment of a Force (or) Torque

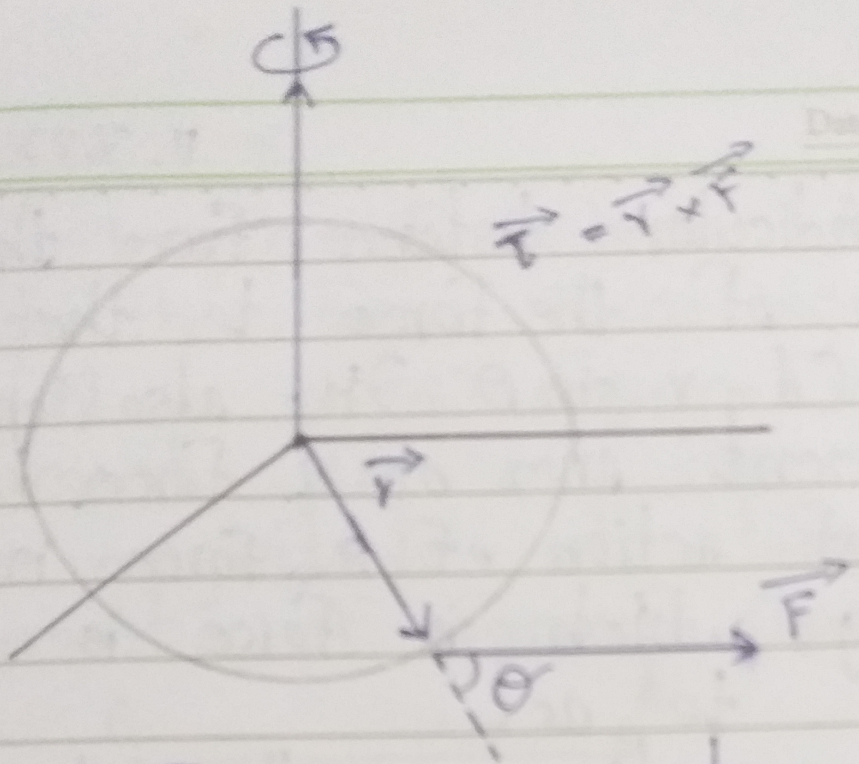
A Force which acts on a pivoted body at a distance From the fulcrum tends to make that body rotate.

The turning effect of a force about a particular fulcrum is measured by the moment of that Force (or) torque.

Torque is a vector quantity and its vector form can be expressed.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The direction of torque is perpendicular to both $\vec{r}$ and $\vec{F}$. The distance From the fulcrum to the point of action of the Force is r and θ is the angle between vectors $\vec{r}$ and $\vec{F}$.



Torque as a vector product of $\vec{r}$ and $\vec{F}$

The magnitude of the moment of force (or) torque τ is defined as the product of the force F and the perpendicular distance d from the line of action of the force to the fulcrum.

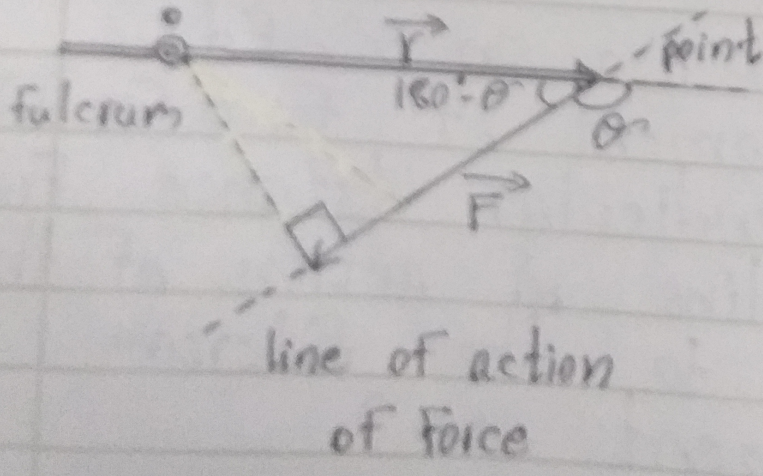
$$\tau = F r \sin\theta = Fd$$

where τ = moment of force (or) Torque
d = perpendicular distance From the line of action of the Force to the fulcrum (moment arm)

Dream it. Do it.

* The perpendicular distance from the line of action of the force to the fulcrum ($d = r \sin\theta$) is also called the moment arm of a force.
* The line of action of a force is a line along which a force is considered to act.
* If a force action on the object is at an angle of 90°, r equals d.

In SI units,
The moment of force (or) torque is measured in newton metre (Nm, which is never written as joule).

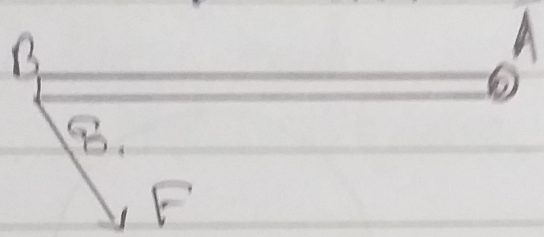


$d = r \sin\theta$

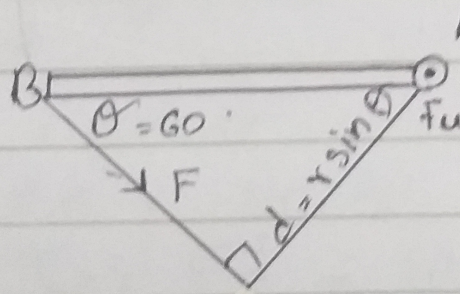
$d = r \sin\theta$

$\sin(180° - \theta) = \sin\theta$

Eg 2.2 The length of a beam AB is 2m and the force F acting at B is 10 N in given figure. Find the ~~reverse~~ moment arm of the force and the magnitude of torque (the moment of Force) about the point A.



To Find the moment arm of the force, we must draw the given diagram as follows.



$F = 10\ N,\ r = 2m$

$d = r \sin\theta$

$= 2 \times \sin 60^\circ$

$= 2 \times 0.866 = 1.732m$

$\tau = Fd = 10 \times 1.732$

$= 17.32\ Nm$

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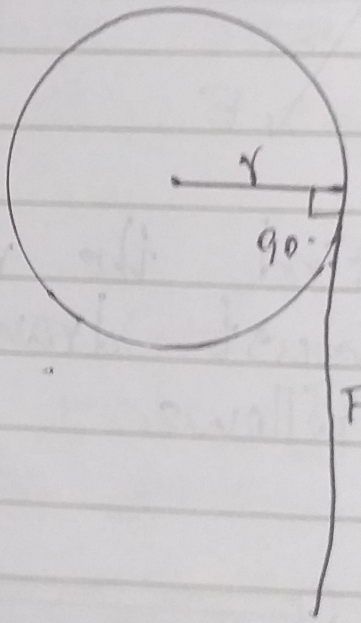
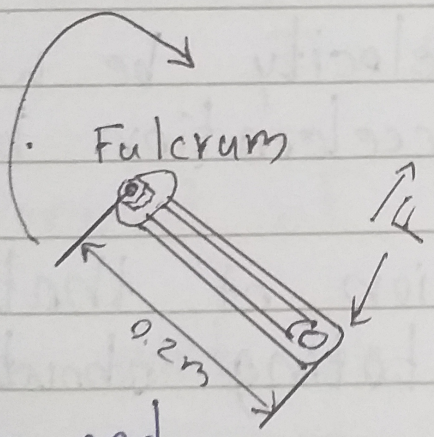
Ex 4. A student pulls down with a force of 40N on a rope that winds around a pulley of radius 0.2 m. Find the torque acting on the pulley.

$F = 40\ N$
$r = 0.2\ m$
$\tau = ?$
$\theta = ?$
$\theta = 90°$



$d = r \sin\theta$
$d = 0.2 \times 1$
$d = 0.2\ m$
$\tau = Fd$
$= 40 \times 0.2$
$= 8\ Nm$

Eg 2.3 In a given Figure a Force of 10 N is applied to a ~~spanner~~ spanner to tighten a nut. The length of the spanner is 0.2 m. What is the moment of a force ~~exact~~ exerted when the Force acts at (i) the end and (ii) the middle of the spanner.



$F = 10N$

$r = 0.2\ m$

$\theta = 90°$

Angle between $\vec{r}$ and $\vec{F} = 90°$

The end,

(i) $d = r \sin\theta$

$= 0.2 \times \sin 90° = 0.2\ m$

$T = Fd = 10 \times 0.2 = 2\ Nm$

(ii) The middle,

$= \frac{r}{2} \sin\theta$

$= \frac{0.2}{2} \times \sin 90°$

$= 0.1 \times 1 = 0.1\ m$, $T = Fd$

$= 10 \times 0.1$

$= 1\ Nm$

Dream it. Do it.

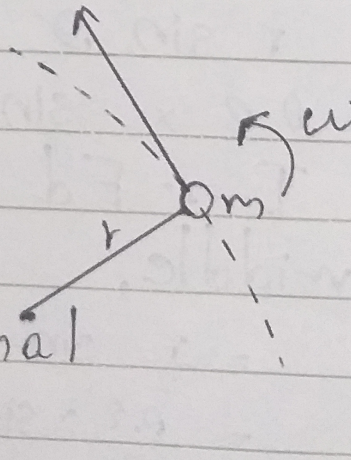
2.3 Relation between Torque and moment of Inertia

Moment of Inertia

Consider a particle of mass m at a distance r from the axis of rotation, being acted upon by a tangential force $\vec{F}$.

let its angular velocity be w and its angular acceleration be α.

Tangential acceleration of that particle ~~rotatio~~ rotating about an axis is $\alpha = r\alpha$.



Dream it. Do it.

According to Newton's second law

$$F = ma = mr\alpha$$

Torque due to Tangential force,

$$\tau = Fr = (mr\alpha)r$$

$$\tau = mr^2\alpha$$

The term mr^2 is the moment of inertia of that particle about the axis (I).

$$\tau = I\alpha.$$

Moment of Inertia of an object

The particles of masses $m_1, m_2, m_3, \ldots, m_i$.

The ~~distancee~~ distances of each particle from the axis of rotation are

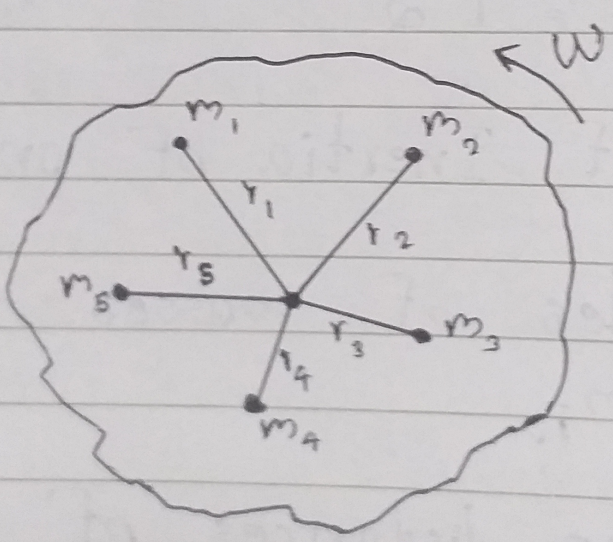
$$r_1, r_2, r_3, \ldots, r_i;$$

The moment of inertia of that object is $m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots + m_ir_i^2$.

The moment of Inertia I can be written as

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_i r_i^2.$$

$$I = \sum_i m_i r_i^2.$$



A rigid object rotating about a fixed axis with the angular velocity ω

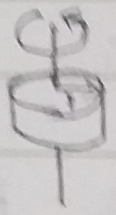
The moment of inertia of an object is defined as the sum of the products obtained by multiplying the mass of each particle in a given object and the square of its distance from the axis.

The moment of inertia is a scalar and the SI unit is $kg\ m^2$.
The moment of inertia must be specified with respect to a ~~choosen~~ chosen axis of rotation.

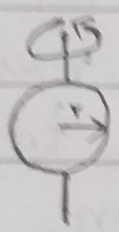
The formulae of moment of inertia for some homogenous objects with respective axis.

Figure Moment of inertia of some homogenous objects

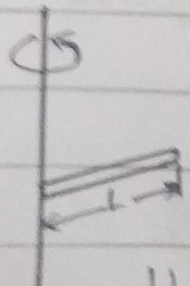
hoop
(or)
cylindrical shell
$I = MR^2$



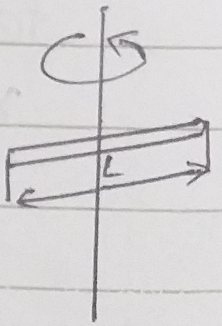
solid cylinder
(or) disk
$I = \frac{1}{2} MR^2$



solid sphere
$I = \frac{2}{5} MR^2$



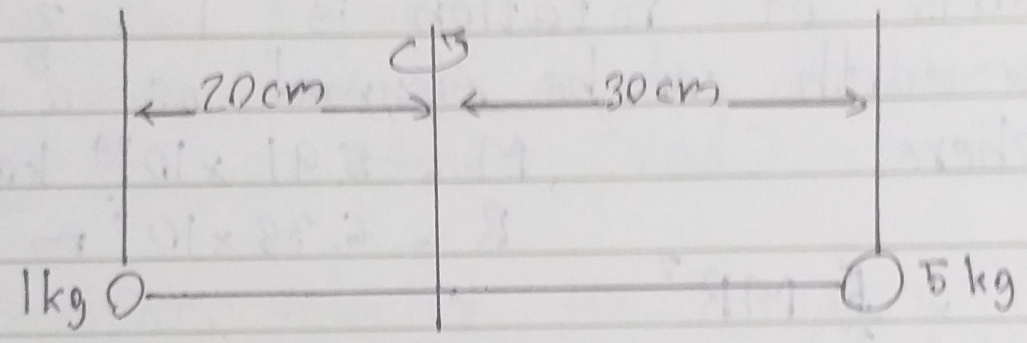
long thin rod with rotation axis through end $I = \frac{1}{3} ML^2$



long thin rod with rotation axis through center
$I = \frac{1}{12} ML^2$

Dream it. Do it.

Eg 2.4 Two balls are connected by a rigid rod of negligible mass. What is its moment of inertia with respect to the axis as shown in figure? The axis is perpendicular to the rod.



$m_1 = 1kg$

$m_2 = 5kg$

$r_1 = 20cm$

$= 20 \times 10^{-2} m$

$r_2 = 30cm$

$= 30 \times 10^{-2} m$

$I = \sum_i m_i r_i^2$

$= m_1 r_1^2 + m_2 r_2^2$

$= 1 \times (20 \times 10^{-2})^2 + 5 \times (30 \times 10^{-2})^2$

$= 400 \times 10^{-4} + 4500 \times 10^{-4}$

$= 4900 \times 10^{-4} = 49 \times 10^{-2}$

Dream it. Do it.

Eg 2.5 Calculate the moment of inertia of the earth about its axis of rotation. Mass of the earth is 5.91×10^{24} kg and its radius is 6.38×10^{6} m. The moment of inertia of solid sphere about its axis of rotation is $I = \frac{2}{5} MR^2$. The earth can be assumed as a solid sphere.

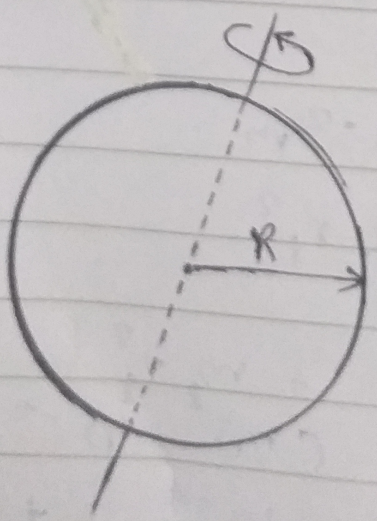
$M = 5.91 \times 10^{24}$ kg

$R = 6.38 \times 10^{6}$ m

$$I = \frac{2}{5} MR^2$$

$$= \frac{2}{5} (5.91 \times 10^{24}) \times (6.38 \times 10^{6})^2$$

$$= 9.68 \times 10^{37} \text{ kgm}^2$$



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5 A simple pendulum with the bob of 0.5 kg and its length of 80 cm is suspended from a rigid clamp. Find the torque produced by the weight of bob about the point of suspension when the pendulum is swung 20° from vertical line.

$m = 0.5$ kg

$r = 80$ cm

$= 80 \times 10^{-2}$ m

$\theta = 20°$

$T = ?$

$F = W$

$W = mg$

$= 0.5 \times 9.8$

$= 4.9$ N

$d = r \sin\theta$

$= 80 \times 10^{-2} \sin 20° = 80 \times 10^{-2} \times 0.342$

$d =$ m

$T = Fd$

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Centre of Gravity

The centre of gravity of a particular object is a point at which all its weight may be considered to act.

2.4. Equilibrium

When the system is in static equilibrium, there is no linear motion ~~and~~ and no rotational motion. Under this condition, the forces on it must be balanced and the moment of the forces on it must also be balanced.

The Principle of Moments.

If an object such as a bar (or) a plank is to be in balance, the total clockwise moment about the fulcrum must equal the total anticlockwise moment.

The examples of the application of this principle are building site crane, beam balance, etc.

Dream it. Do it.

Date 17.7.23

Review Exercise

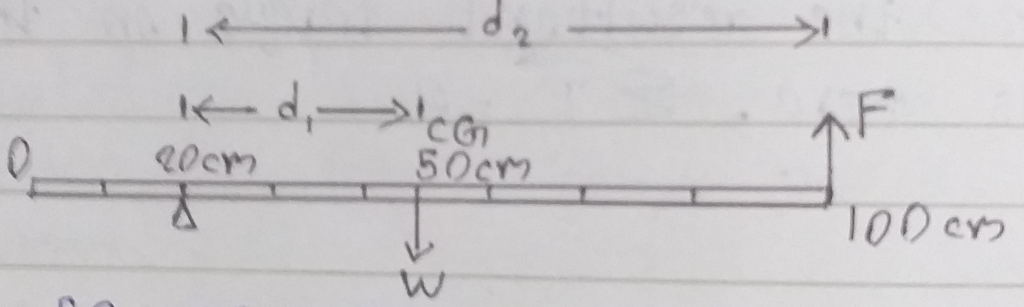
State the conditions of static equilibrium when a body is acted by a number of parallel forces.

Ans The conditions of static equilibrium are
(1) the resultant force on the system is zero, $\Sigma \vec{F} = 0$,
(2) the resultant torque on the system is zero, $\Sigma \vec{\tau} = 0$

Date 16.7.23

*

Eg 2.6 A uniform metre rule weighing ~~4N~~ 4N, pivoted at the 20 cm mark, is supported at the right-hand end at the 100cm mark, by a vertical thread. Find the tension in the thread.



$d_1 = 50 - 20 = 30 \times 10^{-2}$ m

$d_2 = 100 - 20 = 80 \times 10^{-2}$ m

$W = 4$ N, $F = ?$

By the principle of moment,

Total clockwise = Total anticlockwise

$wd_1 = Fd_2$

$4 \times 30 \times 10^{-2} = F \times 80 \times 10^{-2}$

$120 = 80F$

$F = \frac{120}{80}$

$F = 1.5$ N

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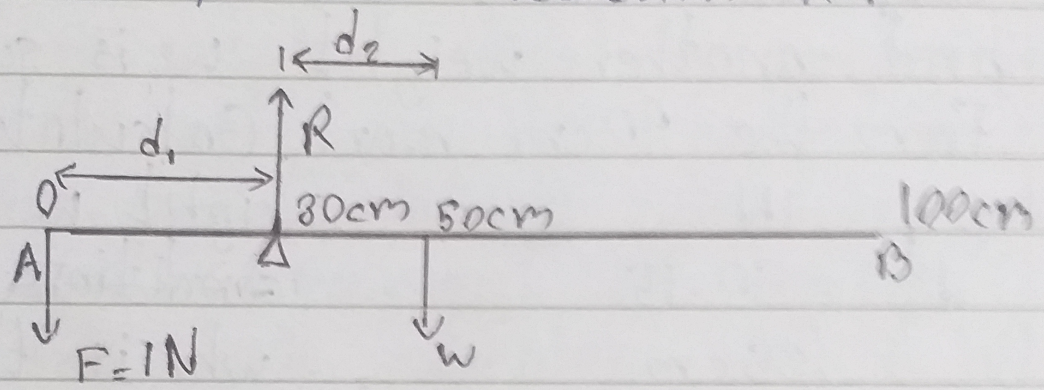
Eg 2.7. If AB is a uniform metre rule which is balanced as shown in the diagram, (i) What is the weight of the metre rule? (ii) What is the reaction R?



(i) $d_1 = 30.0 = 30\ cm = 30 \times 10^{-2}\ m$

$d_2 = 50 - 30 = 20cm = 20 \times 10^{-2}\ m$

$F = 1N$, $W = ?$, $R = ?$

By the principle of moment

Total clockwise = Total anticlockwise

$Wd_2 = Fd_1$

$W \times 20 \times 10^{-2} = 1 \times 30 \times 10^{-2}$

$20W = 30$

$W = \frac{30}{20}$

$W = 1.5\ N$

Dream it. Do it.

Date 18.7.23

8. A uniform beam of 1m balances horizontally about a pivot at its midpoint when a weight of 1N is suspended from the 15cm mark and another weight w is suspended from the 90cm mark. Calculate the weight w. In your ~~condition~~ calculation, why is the weight of beam not included?

$w_1 = 1N$

$d_1 = 50 - 15$

$= 35cm$

$= 35 \times 10^{-2} m$

$d_2 = 90 - 50$

$= 40cm$

$= 40 \times 10^{-2} m$

$w_2 = w = ?$

By the Priniple of moment.

Total ~~ee~~clockwise moment = Total anticlockwise moment

$w_2 d_2 = w_1 d_1$

$w_2 \times (40 \times 10^{-2}) = 1 \times (35 \times 10^{-2})$

$w_2 = \frac{1 \times (35 \times 10^{-2})}{40 \times 10^{-2}}$

$w_2 = \frac{7}{8} = 0.875\ N$

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The weight at the beam not ~~it~~ included because the pivot is at its midpoint which is its centre of gravity.

b) A rod is rotated about a perpendicular axis through at its centre (or) through at one end. (i) Which condition will give the greator moment of inertia?
(ii) How much greater will it be?

(i) M = mass of the rod
L = length of the rod
when the axis is at the centre
moment of inertia $I_1 = \frac{1}{12} ML^2$
when the axis is at one end,
moment of inertia $I_2 = \frac{1}{3} ML^2$
$I_2 > I_1$
When the axis is at one end, it will give greater moment of inertia.

Dream it. Do it.

Date 20.7.23

(ii) $\frac{I_2}{I_1} = \frac{\frac{1}{3}ML^2}{\frac{1}{12}ML^2}$

$\frac{I_2}{I_1} = \frac{1}{3} \times 12^4$

$\frac{I_2}{I_1} = 4$

$I_2 = 4I_1$

11. An object spins with an angular speed ω. If its moment of inertia increases by a factor of 2 without applying an external torque, what will be the its new angular speed?

ω = old angular speed
ω_1 = new angular speed
I = moment of inertia
I_1 = new moment of inertia
$I_1 = 2I$

By the law of conservation of angular momentum,

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Total old angular momentum = Total new angular momentum

$$I\omega = I_1\omega_1$$
$$I\omega = 2I\omega_1$$
$$\omega = 2\omega_1$$
$$\omega = 2\omega_1$$
$$\omega_1 = \frac{\omega}{2}$$

9. A 45 kg girl and a 65 kg boy are sitting on a see-saw in ~~equifibru~~ equilibrium. If the boy is sitting 0.7 m from the fulcrum, where is the girl sitting?

$W_g = 45\,kg \times 9.8\,ms^{-2} = 441\,N$

$W_b = 65\,kg \times 9.8\,ms^{-2} = 637\,N$

$d_b = 0.7\,m, \; d_g = ?$

B

Total Clockwise moment = Total ~~Clockwise~~ Anti-clockwise moment

$W_b d_b = W_g d_g$

$637 \times 0.7 = 441 \times d_g$

$d_g = 1.01\,m$

The girl is sitting 1.01 m from the fulcrum.